Chapter 25

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Electric Potential

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Lecture 06

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Electric Potential Due To Continuous Charge Distributions

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Electric Potential Due to Continuous Charge Distributions

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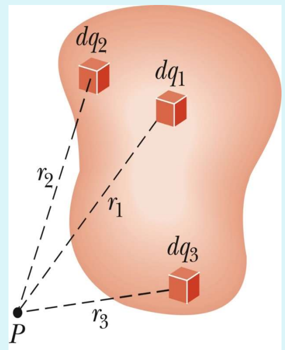
Consider a small charge element dq

Treat it as a point charge. The potential at some point due to this charge element is:

$$dV = k_e \frac{dq}{r}$$

To find the total potential, you need to integrate to include the contributions from all the elements.

$$V = k_e \int \frac{dq}{r}$$



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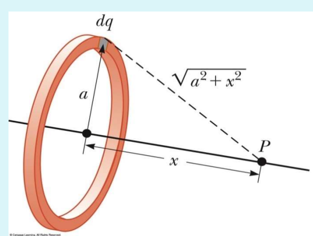
Example 25.5 Electric Potential Due to a Uniformly Charged Ring

A ring of radius (a) carries a uniformly distributed positive total charge (Q). Calculate the electric potential due to the ring at a point (P) lying a distance (x) from its centre along the central axis perpendicular to the plane of the ring.



Answer: The potential is given by

$$V = k_e \int_0^Q \frac{dq}{r} = \frac{k_e Q}{\sqrt{a^2 + x^2}}$$

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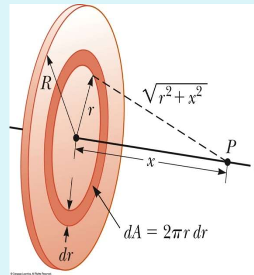
Example 25.6 Electric Potential Due to a Uniformly Charged Disk

A disk of radius (R) has a uniform surface charge density (σ) . Calculate the electric potential at a point (P) that lies along the central perpendicular axis of the disk and a distance (x) from the center of the disk.



Answer: The potential is given by:

$$V = 2\pi k_e \sigma \int_0^R \frac{r\,dr}{\sqrt{x^2+r^2}}$$

$$V = 2\pi k_e \sigma \left(\sqrt{R^2+x^2} - x\right)$$

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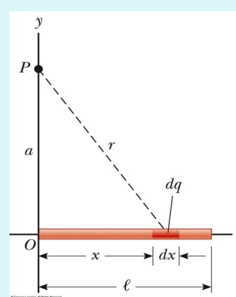
Example 25.7 Electric Potential Due to a Finite Line of Charge

A rod of length (ℓ), located along the x-axis has a total charge (Q) and a uniform linear charge density (λ). Find the electric potential at a point (P) located on the y-axis a distance (a) from the origin.



Answer: The potential is given by:

$$V = k_e \lambda \int_0^{\ell} \frac{dx}{\sqrt{a^2+x^2}}$$

$$V = k_e \lambda \, ln\left(\frac{\ell + \sqrt{a^2+\ell^2}}{a}\right)$$

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Electric Potential Due to a Uniformly Charged Ring

Monday, 1 February, 2021 21:21

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
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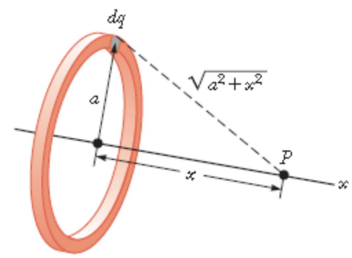
Find an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q.

Answer	
	$V=\frac{k_e}{\sqrt{a^2+x^2}}\int_0^Q dq$
	$V=\frac{k_eQ}{\sqrt{a^2+x^2}}$
At the center, $x=0$	$V=\frac{kQ}{a}$
If $x>>a$:	$V=\frac{kQ}{x}$



the ring is oriented so that its plane is perpendicular to the x axis and its center is at the origin. Notice that the symmetry of the situation means that all the charges on the ring are the same distance from point P. We take point P to be at a distance x from the center of the ring. Express V in terms of the geometry:

$$V=\int\frac{k\,\mathrm{d}q}{r}=k\int\frac{\mathrm{d}q}{\sqrt{a^2+x^2}}$$

Noting that a and x do not vary for an integration over the ring, bring $\sqrt{a^2+x^2}$ in front of the integral sign and integrate over the ring:

$$V=\frac{k}{\sqrt{a^2+x^2}}\int\mathrm{d}q=\frac{kQ}{\sqrt{a^2+x^2}}$$

At the center, $x=0$ and

$$V=\frac{kQ}{a}$$

If P is far away from the ring, $x>>a$:

$$V=\frac{kQ}{x}$$

The ring acts like a point charge.

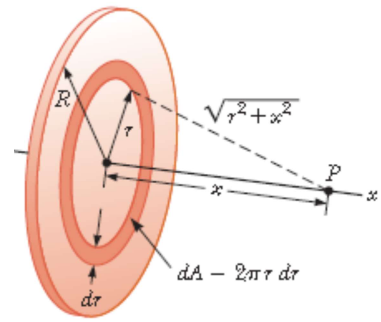
Electric Potential Due to a Uniformly Charged Disk

Monday, 1 February, 2021 21:26

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A uniformly charged disk has radius R and surface charge density σ. Find the electric potential at a point P along the perpendicular central axis of the disk.



Answer	
	$V = k_e\pi\sigma\int_0^R \frac{2r\,dr}{(r^2+x^2)^{1/2}}$
	$V = 2k_e\pi\sigma\left[(x^2+R^2)^{1/2} - x\right]$
At the center, $x = 0$	$V = 2\pi k_e\sigma R = \frac{2kQ}{R}.$
For large values of x, $x \gg R$	$V = \frac{kQ}{x}$
At $x = 0.75\text{R}$	$V = \pi k_e\sigma R = \frac{kQ}{R}$

consider the disk to be a set of concentric rings, we can use our result for the charged ring — which gives the potential due to a ring of radius a — and sum the contributions of all rings making up the disk. Because point P is on the central axis of the disk, symmetry again tells us that all points in a given ring are the same distance from P. Find the amount of charge dq on a ring of radius r and width dr:

$$\mathrm{d}q = \sigma\,\mathrm{d}A = \sigma(2\pi r\,\mathrm{d}r) = 2\pi\sigma r\,\mathrm{d}r$$

Use this result in our result for the charged ring (with a replaced by the variable r and Q replaced by the differential dq) to find the potential due to the ring:

$$\mathrm{d}V = \frac{k\,\mathrm{d}q}{\sqrt{r^2+x^2}} = \frac{2\pi k\sigma r\,\mathrm{d}r}{\sqrt{r^2+x^2}}$$

To obtain the total potential at P, integrate this expression over the limits $r \to 0$ to $r \to R$, noting that x is a constant:

$$V = 2\pi k\sigma\int_0^R \frac{r\,\mathrm{d}r}{\sqrt{r^2+x^2}}$$

Use this integral:

$$\int \frac{x\,\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

Evaluate the integral:

$$V = 2\pi k_e\sigma\left[\sqrt{x^2+R^2} - x\right]$$

At the center, $x = 0$

$$V = 2\pi k_e \sigma R = \frac{2kQ}{R}$$

For large values of x, $x \gg R$, the result above can be evaluated by a series expansion (Taylor expansion) and shown to be equivalent to the electric potential of a point charge Q.

$$(1+a)^n = 1 + na\,, a = \frac{R^2}{x^2}\,, n = \frac{1}{2}$$

$$\left(1+\frac{R^2}{x^2}\right)^{\frac{1}{2}} = 1 + \frac{R^2}{2x^2}$$

$$V = 2\pi k\sigma\left(\sqrt{x^2+R^2} - x\right) = 2\pi k\frac{Q}{\pi R^2}\left(x\left(1+\frac{R^2}{x^2}\right)^{1/2} - x\right)$$

$$V = 2\pi k\frac{Q}{\pi R^2}x\left(\left(1+\frac{R^2}{x^2}\right)^{1/2} - 1\right) = \frac{2kQx}{R^2}\left(1+\frac{R^2}{2x^2} - 1\right)$$

$$V = \frac{kQ}{x}$$

Electric Potential Due to a Finite Line of Charge

Monday, 1 February, 2021 21:30

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

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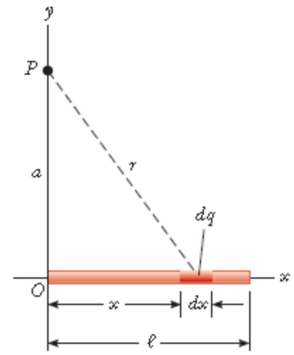
A rod of length ℓ, located along the x axis has a total charge Q and a uniform linear charge density λ. Find the electric potential at a point P located on the y axis a distance a from the origin.



Answer:	
	$V = k_e\lambda \int_0^{\ell} \frac{dx}{(a^2+x^2)^{1/2}}$
	$V = \frac{k_e Q}{\ell} ln\left(\frac{\ell + \sqrt{a^2+\ell^2}}{a}\right)$
If $L \ll a$	$V \cong \frac{k_e Q}{a}$

The potential at P due to every segment of charge on the rod is positive because every segment carries a positive charge. Notice that we have no symmetry to appeal to here, but the simple geometry should make the problem solvable.

The rod lies along the x axis, dx is the length of one small segment, and dq is the charge on that segment. Because the rod has a charge per unit length λ, the charge dq on the small segment is

$$dq = \lambda\, dx$$

Find the potential at P due to one segment of the rod at an arbitrary position x:

$$dV = \frac{k\, dq}{r} = k\lambda \frac{dx}{\sqrt{x^2+a^2}}$$

Find the total potential at P by integrating this expression over the limits $x = 0$ to $x = L$:

$$V = k\lambda \int_0^L \frac{dx}{\sqrt{x^2+a^2}}$$

Noting that k and $\lambda = Q/L$, are constants and can be removed from the integral, use the integral:

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

Evaluate the integral:

$$V = \frac{k_e Q}{\ell}\left[ln\left(\ell + \sqrt{a^2+\ell^2}\right) - ln(a)\right] = \frac{k_e Q}{\ell} ln\left(\frac{\ell + \sqrt{a^2+\ell^2}}{a}\right)$$

If $L << a$, the potential at P should approach that of a point charge because the rod is very short compared to the distance from the rod to P. By using a series expansion for the natural logarithm ($\ln(1 + x) \approx x$):

$$V = \frac{k_e Q}{\ell} ln\left(\frac{\ell + \sqrt{a^2 + \ell^2}}{a}\right)$$

$$V = \frac{k_e Q}{\ell} ln\left(\frac{\ell}{a} + \frac{\sqrt{a^2 + \ell^2}}{a}\right) = \frac{k_e Q}{\ell} ln\left(\frac{\ell}{a} + \frac{a\sqrt{1 + \frac{\ell^2}{a^2}}}{a}\right)$$

$$V \cong \frac{k_e Q}{\ell} ln\left(\frac{\ell}{a} + 1\right) \cong \frac{k_e Q}{\ell}\frac{\ell}{a} \cong \frac{k_e Q}{a}$$

Electric Potential Due to a Uniformly Charged Sphere

Sunday, 28 March, 2021 21:08

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

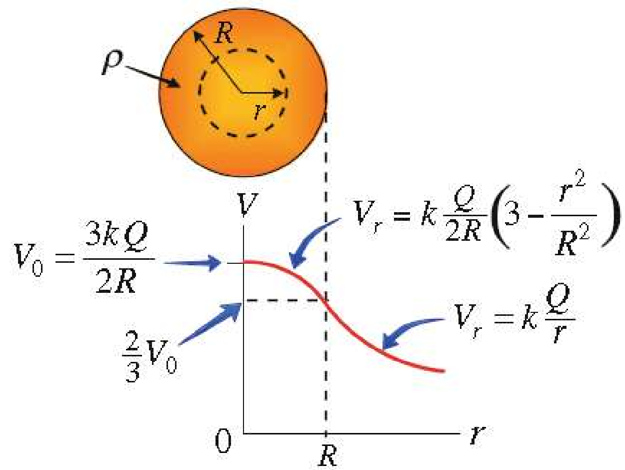
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.

H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

An insulating solid sphere of radius R has a uniform volume charge density ρ and carries a total positive charge Q.

- Find the electric potential at a point outside the sphere.
- Find the electric potential at a point inside the sphere.

Answers	
When $r > R$:	$V = \frac{k_e Q}{r}$
When $r < R$:	$V = \frac{k_e Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$
At $r = 0$	$V_0 = \frac{3kQ}{2R}$



First, we find the electric potential in the region $r \geq R$ by using the electric field obtained previously. In this region, we found that $\vec{E}$ is radial and has a magnitude:

$$E = \frac{kQ}{r^2}$$

This is the same as the electric field due to a point charge, and hence the electric potential at any point of radius r in this region is given by:

$$V = \frac{kQ}{r}$$

The potential at a point on the surface of the sphere $(r = R)$ is:

$$V = \frac{kQ}{R}$$

In the region $0 \leq r \leq R$ inside the sphere, we use the result of the electric field obtained previously. In this region, we found that $\vec{E}$ is radial and has a magnitude:

$$E = \frac{kQ}{R^3} r$$

For a point that has a radius r in the region $0 \leq r \leq R$, we can find the potential difference between this point and any point on the surface with a radius R by using,

$$\vec{E} \cdot \mathrm{d}\vec{s} = E\,\mathrm{d}r$$

Thus:

$$V_r - V_R = -\int_R^r \vec{E} \cdot \mathrm{d}\vec{s} = -\int_R^r E\,\mathrm{d}r = \int_r^R \frac{kQ}{R^3} r\,\mathrm{d}r$$

$$V_r - V_R = \frac{kQ}{2R^3}\,(r^2)|_r^R = \frac{kQ}{2R^3}\,(R^2 - r^2)$$

$$V_r - V_R = \frac{k_e Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

At the center, $r = 0$, we have,

$$V_0 = \frac{3kQ}{2R}$$